MULTIMEDIA UNIVERSITY

STUDENT ID NO

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# MULTIMEDIA UNIVERSITY

# FINAL EXAMINATION

**TRIMESTER 3, 2022/2023 (TERM ID 2230)**

## PCL0016 – CALCULUS

(ALL Sections/Groups)

6 OCTOBER 2023
9:00 A.M - 11:00 A.M.
(2 Hours)

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**INSTRUCTIONS TO STUDENT**

1. This question paper consists of 5 pages including cover page with FOUR questions and an appendix.

2. Attempt **ALL FOUR** questions and the distribution of the marks for each question is given.

3. Please write all your answers in the Answer Booklet provided.

**Question 1 (25 marks)**

(a) Evaluate the following limits:

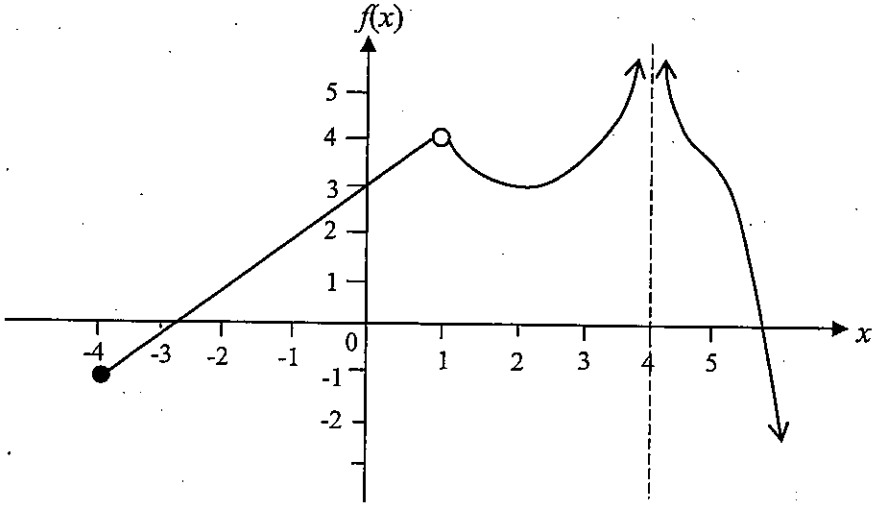
(i) $\lim_{x\to 2}\left(x^3+2x+7\right)$ [2 marks]

(ii) $\lim_{x\to 4}\dfrac{x^2-2x-8}{x-4}$ by factorization method. [4 marks]

(iii) $\lim_{x\to\infty}\dfrac{x^5-2x^2+3}{2x^5}$. [3 marks]

(iv) $\lim_{x\to 0}\dfrac{\sin 5x}{\sin 7x}$ (Hint: $\lim_{x\to 0}\dfrac{\sin x}{x}=1$). [3 marks]

(b) The graph of a function $f(x)$ is given below.




Determine the values of the following limits if they exist. If they do not exist, justify the reason.

(i) $\lim_{x\to 0} f(x)$ [4 marks]

(ii) $\lim_{x\to 4} f(x)$ [4 marks]

(c) Find constant $a$ so that the given function will be continuous for all $x$.

$$f(x)=\begin{cases} ax+3 & \text{if } x>5 \\ x^2+2x+1 & \text{if } x\le 5 \end{cases}$$ [5 marks]

**Continued.....**

**Question 2 (25 marks)**

(a) Evaluate $\frac{dy}{dx}$ for the following functions:

(i) $y=\left(\frac{x}{x-3}\right)^3$ by applying the chain rule. [4 marks]

(ii) $y=\frac{x^4(x-3)^{\frac{1}{2}}}{(x-6)^3}$ by using logarithmic differentiation. [6 marks]

(b) Evaluate the following integrals:

(i) $\int_1^4 3x^4+2x^2-9\ dx+\int_0^{\pi}\cos x\ dx$ [5 marks]

(ii) $\int 12x^2\sqrt{4x^3+15}\ dx$ by using integration by substitution. [4 marks]

(iii) $\int\frac{x^2+3x-1}{(x+1)(x+2)}dx$ by using partial fraction decomposition. [6 marks]

**Question 3 (25 marks)**

(a) Given $f(x)=x^3-3x+1$ is define on the interval $[0,3]$.

(i) Differentiate $f(x)$ and find the critical number of the function. Then, find the values of $f$ at the critical number. [5 marks]

(ii) Evaluate the endpoints. Hence, determine the absolute maximum and minimum values of the function. [4 marks]

(b) A force of 50 N is required to stretch a spring 20 cm beyond its natural length. How much work will be required if the spring is stretched 30 cm from its natural length? [5 marks]

(c) A region is bounded by $y=\sqrt{x}$, $y=3$ and the $y$-axis. Sketch the bounded region and find the following:

(i) Area of the bounded region. [7 marks]

(ii) By using the disk method, find the volume of the solid generated by revolving the shaded region about the $y$-axis. [4 marks]

**Continued.....**

**Question 4 (25 marks)**

(a) Solve the differential equation $\frac{dy}{dx} = 6x^2 y$ given the initial condition $y(2) = 1$ by separation of variables method. [7 marks]

(b) Solve the linear differential equation $\frac{dy}{dx} + 4xy = 4x$. [8 marks]

(c) Find the solutions of the following second order differential equations.
$y'' + 7y' = 0, \quad y(0) = 1, y'(0) = 2$. [10 marks]

**End of Paper**

## *APPENDIX*
## Applications of integration formulas

**Area between curves:**

$$A = \int_a^b [f(x) - g(x)]\, dx$$

**Volume of solid:**

Disk: $V = \int_a^b \pi[f(x)]^2\, dx$ Washer: $V = \int_a^b \pi[f(x)]^2 - [g(x)]^2\, dx$

Cylindrical shell: $V = \int_a^b 2\pi x[f(x)]\, dx$

**Length of a plane curve:**

$$L = \int_a^b \sqrt{1 + [f'(x)]^2}\, dx$$

**Work:**

$$W = \int_a^b F(x)\, dx$$

Hooke's law: $F = kx$

**Average value:**

$$f_{ave} = \frac{1}{b-a}\int_a^b f(x)\, dx$$